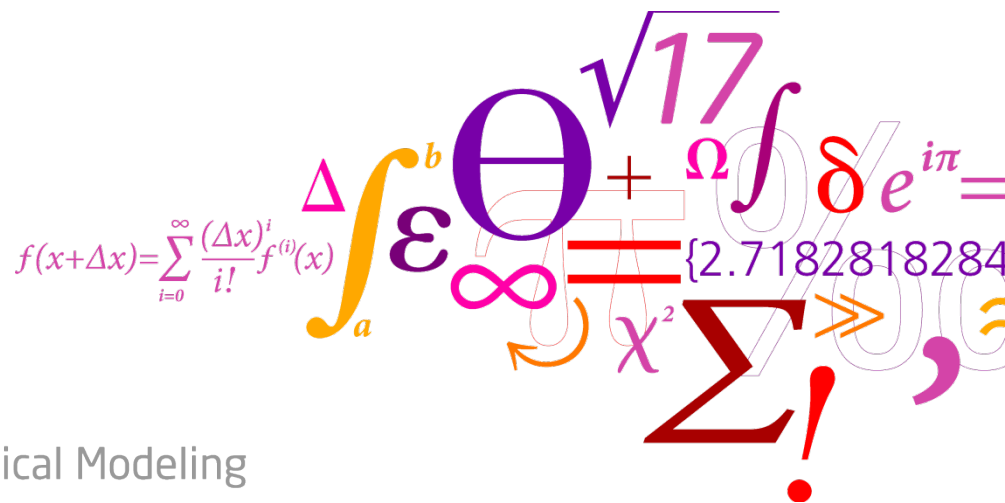


Tools for decision making under uncertainty in electricity markets

Lecture 2: “Stochastic optimization: formal concepts”

Juan Miguel Morales
Salvador Pineda



Example

Certain information			
Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Price (\$/unit)	60	40	10

Example

Certain information			
Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Price (\$/unit)	60	40	10
Demand (low)	50	20	200

Example

x_l = number of lumber items

x_f = number of labor hours for finishing

x_c = number of labor hours for carpentry

y_d = number of desks produced

y_t = number of tables produced

y_c = number of chairs produced

Example

$$\text{Max } 60y_d + 40y_t + 10y_c - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_d \leq 50$$

$$y_t \leq 20$$

$$y_c \leq 200$$

$$8y_d + 6y_t + y_c - x_l \leq 0$$

$$4y_d + 2y_t + 1.5y_c - x_f \leq 0$$

$$2y_d + 1.5y_t + 0.5y_c - x_c \leq 0$$

Certain information			
Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Price (\$/unit)	60	40	10
Demand	50	20	200

Example

Demand		Low
Resource quantities	Lumber	520
	Finishing	240
	Carpentry	130
Production quantities	Desk	50
	Tables	20
	Chairs	0
Profit (\$)		1124

Example

Certain information			
Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Price (\$/unit)	60	40	10
Demand (medium)	150	110	225

Example

Demand	
Resource quantities	Lumber
	Finishing
	Carpentry
Production quantities	Desk
	Tables
	Chairs
Profit (\$)	

Medium
1860
820
465
150
110
0
3982

Example

Certain information			
Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Price (\$/unit)	60	40	10
Demand (high)	250	250	500

Example

Demand	
Resource quantities	Lumber
	Finishing
	Carpentry
Production quantities	Desk
	Tables
	Chairs
Profit (\$)	

High
3500
1500
875
250
250
0
7450

Example

Demand		Low	Medium	High
Resource quantities	Lumber	520	1860	3500
	Finishing	240	820	1500
	Carpentry	130	465	875
Production quantities	Desk	50	150	250
	Tables	20	110	250
	Chairs	0	0	0
Profit (\$)		1124	3982	7450

Example

Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Demand	150	125	300
Price (\$/unit)	60	40	10
Cost (\$/unit)	42.4	27.8	10.6
Profit (\$/unit)	17.6	12.2	-0.6

Example

Certain information			
Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Price (\$/unit)	60	40	10

Unknown information					
		Desk	Table	Chair	Prob
Demand	Low	50	20	200	0.3
	Medium	150	110	225	0.4
	High	250	250	500	0.3

Example

- If some information is uncertain, what would be the best decision?
 - A) make the decisions considering the low demand. If the demand happens to be higher, we will not satisfy it.
 - B) make the decisions considering the medium demand. If the demand is higher, we will not satisfy it. If the demand is lower, we will lose some money due to extra resources.
 - C) make the decisions considering the high demand. If demand happens to be lower, we will lose money for having more resources than needed.
 - D) make the decisions based on the average demand. If the demand is higher, we will not satisfy it. If the demand is lower, we will lose some money due to extra resources.

Example

A) make the decisions considering the low demand.

Strategy		A)		
Resource quantities	Lumber	520		
	Finishing	240		
	Carpentry	130		
DEMAND		Low	Medium	High
Production quantities	Desk	50	50	50
	Tables	20	20	20
	Chairs	0	0	0
Profit (\$)		1124	1124	1124
Expected profit (\$)		1124		

Example

- B) make the decisions considering the medium demand.

Strategy		B)		
Resource quantities	Lumber	1860		
	Finishing	820		
	Carpentry	465		
DEMAND		Low	Medium	High
Production quantities	Desk	50	150	150
	Tables	20	110	110
	Chairs	200	0	0
Profit (\$)		-3618	3982	3982
Expected profit (\$)		1702		

Example

C) make the decisions considering the high demand.

Strategy				C)
Resource quantities	Lumber			3500
	Finishing			1500
	Carpentry			875
DEMAND		Low	Medium	High
Production quantities	Desk	50	150	250
	Tables	20	110	250
	Chairs	200	225	0
Profit (\$)		-11750	-1900	7450
Expected profit (\$)		-2050		

Example

D) make the decisions based on the average demand.

Strategy					D)
Resource quantities	Lumber				1950
	Finishing				850
	Carpentry				487.5
DEMAND		Low	Medium	High	
Production quantities	Desk	50	150	150	
	Tables	20	110	125	
	Chairs	200	20	0	
Profit (\$)		-4035	3765	4165	
Expected profit (\$)		1545			

Example

Strategy		A)	B)	C)	D)
Resource quantities	Lumber	520	1860	3500	1950
	Finishing	240	820	1500	850
	Carpentry	130	465	875	487.5
Expected Profit (\$)		1124	1702	-2050	1545

Note that each strategy has been determined considering just one scenario (deterministic)

Example

- If some information is uncertain, what would be the best decision?
 - A) make the decisions considering the low demand. If the demand happens to be higher, we will not satisfy it.
 - B) make the decisions considering the medium demand. If the demand is higher, we will not satisfy it. If the demand is lower, we will lose some money due to extra resources.
 - C) make the decisions considering the high demand. If demand happens to be lower, we will lose money for having more resources than needed.
 - D) make the decisions based on the average demand. If the demand is higher, we will not satisfy it. If the demand is lower, we will lose some money due to extra resources.
 - E) solve the stochastic problem.

Example

- How do we convert from deterministic to stochastic?
 - 1) Which is the uncertain parameter?

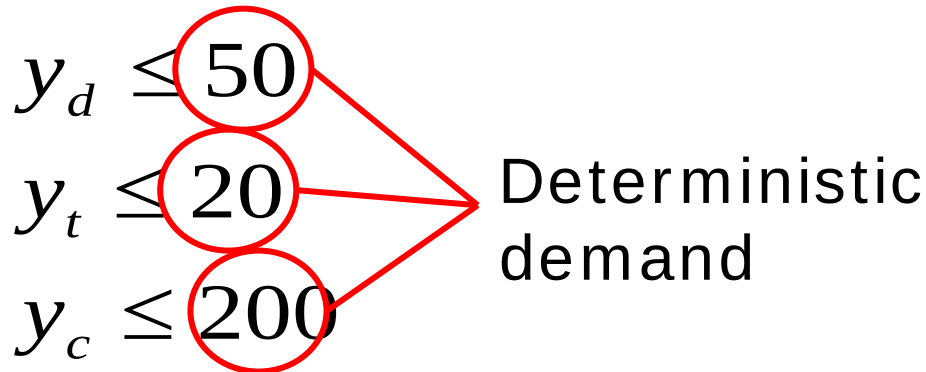
Example

$$\text{Max } 60y_d + 40y_t + 10y_c - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$\begin{aligned} y_d &\leq 50 \\ y_t &\leq 20 \\ y_c &\leq 200 \end{aligned}$$

Deterministic demand



$$8y_d + 6y_t + y_c - x_l \leq 0$$

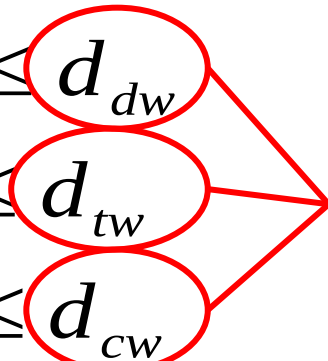
$$4y_d + 2y_t + 1.5y_c - x_f \leq 0$$

$$2y_d + 1.5y_t + 0.5y_c - x_c \leq 0$$

Example

$$\text{Max } 60y_d + 40y_t + 10y_c - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$\begin{aligned} y_d &\leq d_{dw} \\ y_t &\leq d_{tw} \\ y_c &\leq d_{cw} \end{aligned}$$


Stochastic demand

$$8y_d + 6y_t + y_c - x_l \leq 0$$

$$4y_d + 2y_t + 1.5y_c - x_f \leq 0$$

$$2y_d + 1.5y_t + 0.5y_c - x_c \leq 0$$

Example

- How do we convert from deterministic to stochastic?
 - 1) Which is the uncertain parameter?
 - 2) Which decisions does not depend on the scenario realization?

Example

$$\text{Max } 60y_d + 40y_t + 10y_c - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_d \leq d_{dw}$$

$$y_t \leq d_{tw}$$

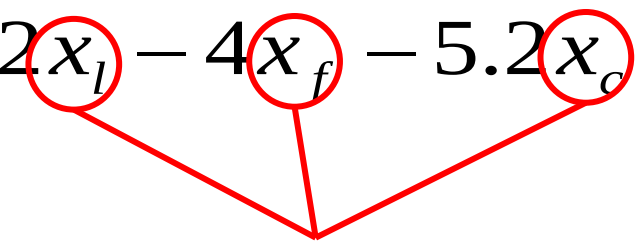
$$y_c \leq d_{cw}$$

$$8y_d + 6y_t + y_c - x_l \leq 0$$

$$4y_d + 2y_t + 1.5y_c - x_f \leq 0$$

$$2y_d + 1.5y_t + 0.5y_c - x_c \leq 0$$

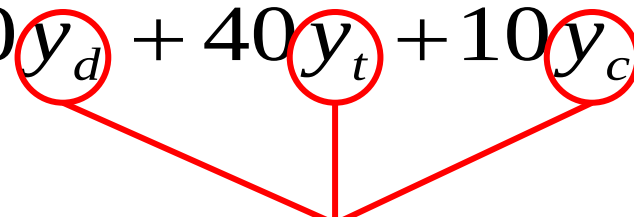
Resources



Example

- How do we convert from deterministic to stochastic?
 - 1) Which is the uncertain parameter?
 - 2) Which decisions does not depend on the scenario realization?
 - 3) Which decisions can be made once the uncertain parameters are known?

Example

$$\text{Max } 60y_d + 40y_t + 10y_c - 2x_l - 4x_f - 5.2x_c$$


s.t.

$$y_d \leq d_{dw} \quad \text{Production}$$

$$y_t \leq d_{tw}$$

$$y_c \leq d_{cw}$$

$$8y_d + 6y_t + y_c - x_l \leq 0$$

$$4y_d + 2y_t + 1.5y_c - x_f \leq 0$$

$$2y_d + 1.5y_t + 0.5y_c - x_c \leq 0$$

Example

$$\text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c$$

s.t. Expected profit

$$y_d \leq d_{dw}$$

$$y_t \leq d_{tw}$$

$$y_c \leq d_{cw}$$

$$8y_d + 6y_t + y_c - x_l \leq 0$$

$$4y_d + 2y_t + 1.5y_c - x_f \leq 0$$

$$2y_d + 1.5y_t + 0.5y_c - x_c \leq 0$$

Example

$$\text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

Example

E) solve the stochastic problem.

DEMAND					E)
Resource quantities	Lumber				1300
	Finishing				540
	Carpentry				325
DEMAND		Low	Medium	High	
Production quantities	Desk	50	80	80	
	Tables	20	110	110	
	Chairs	200	0	0	
Profit (\$)		-650	2750	2750	
Expected profit (\$)		1730			

Example

Strategy		A)	B)	C)	D)	E)
Resource quantities	Lumber	520	1860	3500	1950	1300
	Finishing	240	820	1500	850	540
	Carpentry	130	465	875	487.5	325
Expected Profit (\$)		1124	1702	-2050	1545	1730

Stochastic
programming
works!

Formal concepts

- *Stochastic linear programs* are linear programs in which some problem data may be considered uncertain.

Our example? 

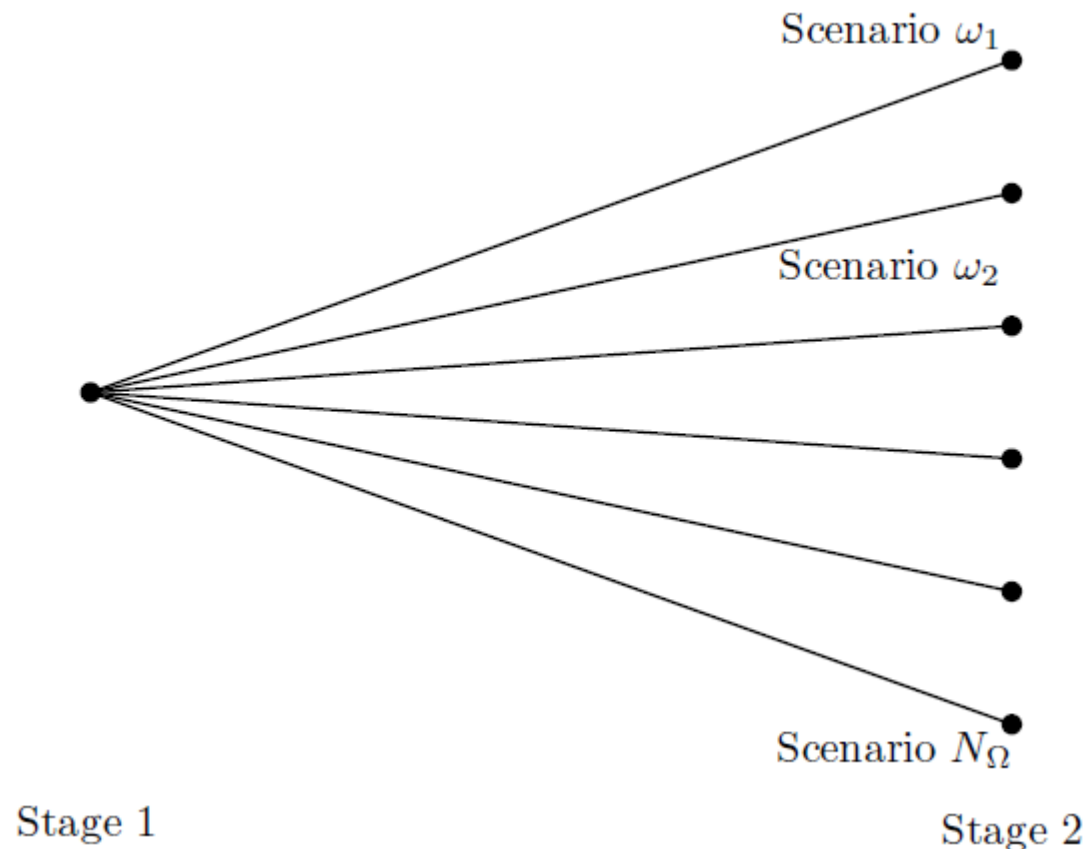
- *Recourse programs* are those in which some decisions or recourse actions can be taken after uncertainty is disclosed.

Our example? 

- first-stage decisions: resources quantities
- second stage decisions: production
- This course: two-stage stochastic linear programs with recourse

Formal concepts

- *Two-stage decision pattern*



Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

Demand		Low	Medium	High
Resource quantities	Lumber	520	1860	3500
	Finishing	240	820	1500
	Carpentry	130	465	875
Production quantities	Desk	50	150	250
	Tables	20	110	250
	Chairs	0	0	0
Profit (\$)		1124	3982	7450

Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

Demand		Low	Medium	High
Resource quantities	Lumber	520	1860	3500
	Finishing	240	820	1500
	Carpentry	130	465	875
Production quantities	Desk	50	150	250
	Tables	20	110	250
	Chairs	0	0	0
Profit (\$)		1124	3982	7450
P.I. "expected" profit (\$)		4165		

Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

P.I. “expected” profit (\$)	4165
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Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

DEMAND					E)
Resource quantities	Lumber				1300
	Finishing				540
	Carpentry				325
DEMAND		Low	Medium	High	
Production quantities	Desk	50	80	80	
	Tables	20	110	110	
	Chairs	200	0	0	
Profit (\$)		-650	2750	2750	
S.P. Expected profit (\$)		1730			

Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

S.P. Expected profit (\$)	1730
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Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

P.I. “expected” profit (\$)	4165
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S.P. Expected profit (\$)	1730
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$$\text{EVPI} = 4165 - 1730 = \$2435$$

Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

$$\text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

$$\begin{aligned}
 & \text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_{lw} - 4x_{fw} - 5.2x_{cw} \\
 & \text{s.t.} \\
 & \quad y_{dw} \leq d_{dw} \\
 & \quad y_{tw} \leq d_{tw} \\
 & \quad y_{cw} \leq d_{cw} \\
 & \quad 8y_{dw} + 6y_{tw} + y_{cw} - x_{lw} \leq 0 \\
 & \quad 4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_{fw} \leq 0 \\
 & \quad 2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_{cw} \leq 0
 \end{aligned}
 \quad \left. \vphantom{\sum_{w=1}^{N_{\Omega}}} \right\} \text{WS}$$

Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

$$\text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

} *RP*

Formal concepts

- *Expected Value of Perfect Information* (EVPI) measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

$$EVPI = WS - RP$$

$$WS = E_{\xi} [\min_x z(x, \xi)] = E_{\xi} z(\bar{x}(\xi), \xi)$$

$$RP = \min_x E_{\xi} z(x, \xi)$$

Formal concepts

- *Value of Stochastic Solution* (VSS) compares the solution between the average problem and the recourse problem.

Strategy		A)	B)	C)	D)	E)
Resource quantities	Lumber	520	1860	3500	1950	1300
	Finishing	240	820	1500	850	540
	Carpentry	130	465	875	487.5	325
Expected Profit (\$)		1124	1702	-2050	1545	1730

$$VSS = 100 * (1730 - 1545) / 1730 = 10.7\%$$

Formal concepts

- *Value of Stochastic Solution* (VSS) compares the solution between the average problem and the recourse problem.

$$\text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

Formal concepts

- *Value of Stochastic Solution* (VSS) compares the solution between the average problem and the recourse problem.

$$\begin{array}{ll}
 \text{Max } 60y_d + 40y_t + 10y_c - 2x_l - 4x_f - 5.2x_c & \\
 \text{s.t.} & \\
 y_d \leq \bar{d}_{dw} & \\
 y_t \leq \bar{d}_{tw} & \\
 y_c \leq \bar{d}_{cw} & \\
 8y_d + 6y_t + y_c - x_l \leq 0 & \\
 4y_d + 2y_t + 1.5y_c - x_f \leq 0 & \\
 2y_d + 1.5y_t + 0.5y_c - x_c \leq 0 &
 \end{array}
 \left. \vphantom{\begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array}} \right\} (\bar{x}_l, \bar{x}_f, \bar{x}_c)$$

Formal concepts

- *Value of Stochastic Solution* (VSS) compares the solution between the average problem and the recourse problem.

$$\begin{aligned}
 & \text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2\bar{x}_l - 4\bar{x}_f - 5.2\bar{x}_c \\
 & \text{s.t.} \\
 & \quad y_{dw} \leq d_{dw} \\
 & \quad y_{tw} \leq d_{tw} \\
 & \quad y_{cw} \leq d_{cw} \\
 & \quad 8y_{dw} + 6y_{tw} + y_{cw} - \bar{x}_l \leq 0 \\
 & \quad 4y_{dw} + 2y_{tw} + 1.5y_{cw} - \bar{x}_f \leq 0 \\
 & \quad 2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - \bar{x}_c \leq 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} \text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2\bar{x}_l - 4\bar{x}_f - 5.2\bar{x}_c \\ \text{s.t.} \\ y_{dw} \leq d_{dw} \\ y_{tw} \leq d_{tw} \\ y_{cw} \leq d_{cw} \\ 8y_{dw} + 6y_{tw} + y_{cw} - \bar{x}_l \leq 0 \\ 4y_{dw} + 2y_{tw} + 1.5y_{cw} - \bar{x}_f \leq 0 \\ 2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - \bar{x}_c \leq 0 \end{aligned}} \right\} EEV$$

Formal concepts

- *Value of Stochastic Solution* (VSS) compares the solution between the average problem and the recourse problem.

$$\begin{aligned}
 & \text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c \\
 & \text{s.t.} \\
 & \quad y_{dw} \leq d_{dw} \\
 & \quad y_{tw} \leq d_{tw} \\
 & \quad y_{cw} \leq d_{cw} \\
 & \quad 8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0 \\
 & \quad 4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0 \\
 & \quad 2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} \text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c \\ \text{s.t.} \\ y_{dw} \leq d_{dw} \\ y_{tw} \leq d_{tw} \\ y_{cw} \leq d_{cw} \\ 8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0 \\ 4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0 \\ 2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0 \end{aligned}} \right\} RP$$

Formal concepts

- *Value of Stochastic Solution* (VSS) compares the solution between the average problem and the recourse problem.

$$VSS = EEV - RP$$

$$VSS(\%) = 100 \frac{EVV - RP}{RP}$$

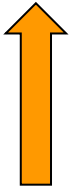
$$EEV = E_{\xi} z(\bar{x}(\bar{\xi}), \xi)$$

$$RP = \min_x E_{\xi} z(x, \xi)$$

Formal concepts

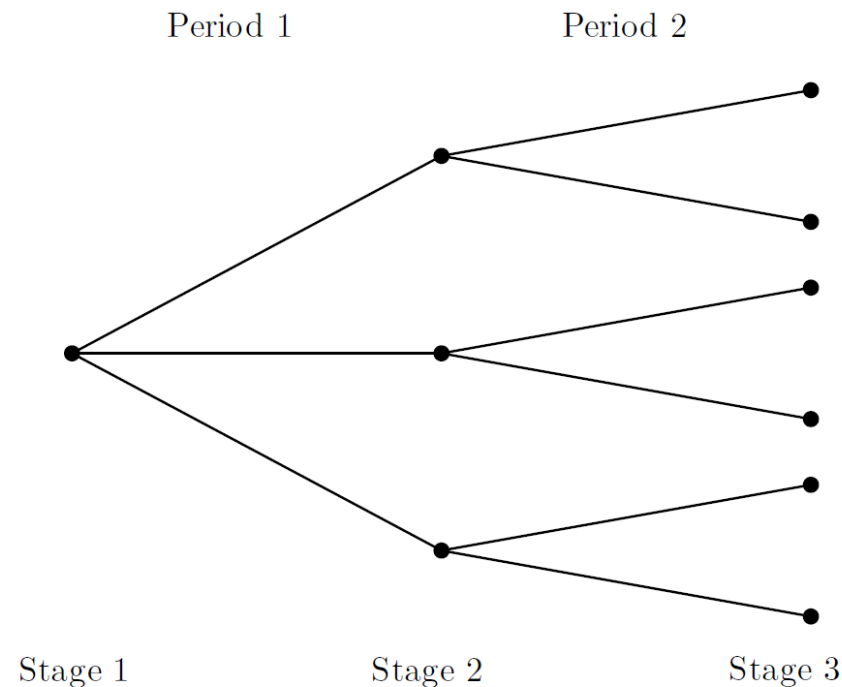
- *Multi-stage stochastic problems* deal with models in which this “decide-observe-decide” pattern is repeated more than one.

M	T	W	T	F	S	S
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Formal concepts

- *Multi-stage stochastic problems* deal with models in which this “decide-observe-decide” pattern is repeated more than one.

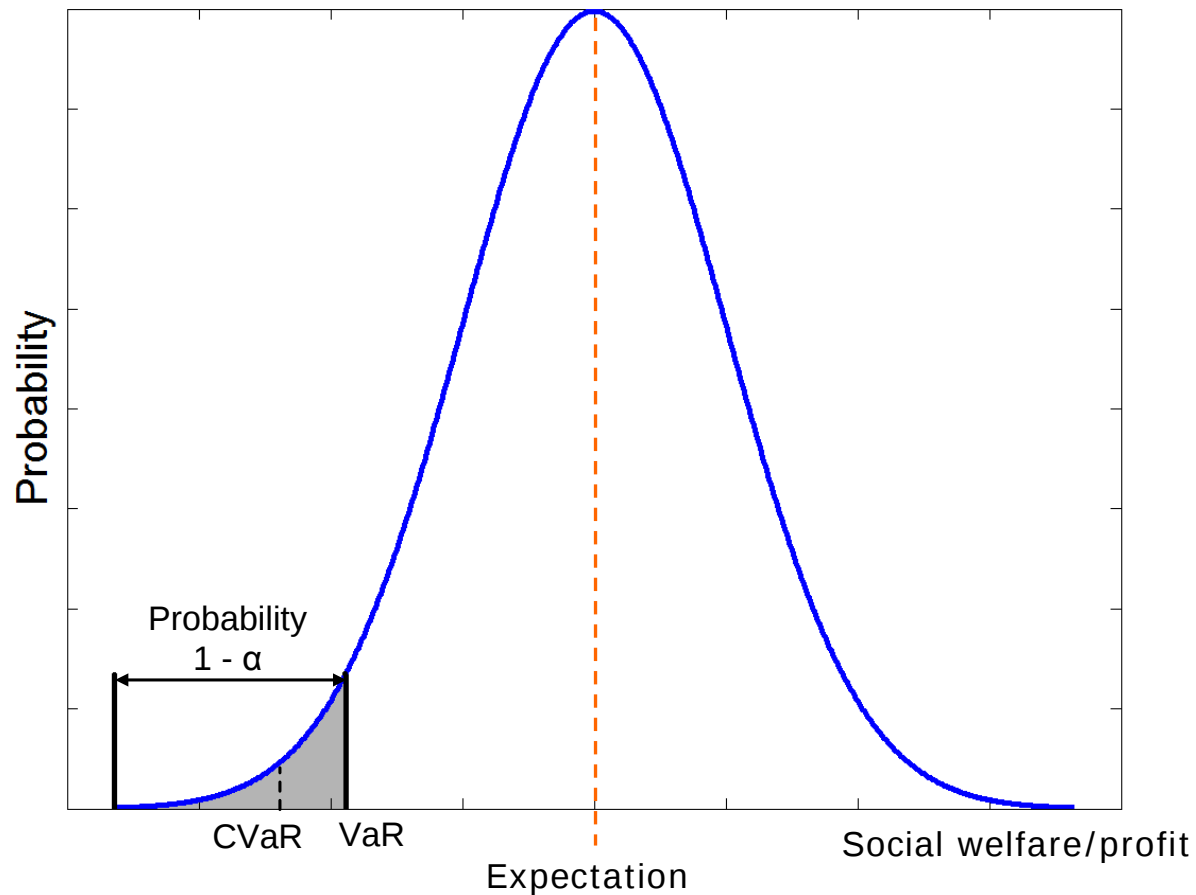


Formal concepts

- *Multi-stage stochastic problems* deal with models in which this “decide-observe-decide” pattern is repeated more than one.
 - Modify the scenario generation procedure
 - Non-anticipativity constraints to model the observe-decide-observe pattern.

Formal concepts

- *Risk aversion*



Formal concepts

- *Risk aversion*

$$\text{Max} \sum_{w=1}^{N_{\Omega}} \pi_w (60y_{dw} + 40y_{tw} + 10y_{cw}) - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

Formal concepts

- *Risk aversion*

$$\text{Max } E(\text{profit}_w)$$

s.t.

$$\text{profit}_w = 60y_{dw} + 40y_{tw} + 10y_{cw} - 2x_l - 4x_f - 5.2x_c$$

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

Formal concepts

- *Risk aversion*

$$\text{Max } CVaR_{\alpha}(\text{profit}_w)$$

s.t.

$$\text{profit}_w = 60y_{dw} + 40y_{tw} + 10y_{cw} - 2x_l - 4x_f - 5.2x_c$$

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

Formal concepts

- $\alpha = 0.5 \rightarrow$ we maximize the 50% worst profit

Strategy	A)	B)	C)	D)	E)
Profit (low demand)	1124	-3618	-11750	-4035	-650
Profit (medium demand)	1124	3982	-1900	3765	2750
Profit (high demand)	1124	3982	7450	4165	2750
Expected Profit (\$)	1124	1702	-2050	1545	1730

Formal concepts

- *Risk aversion* (strategy F)

$$\text{Max } CVaR_{\alpha=0.5}(\text{profit}_w)$$

s.t.

$$\text{profit}_w = 60y_{dw} + 40y_{tw} + 10y_{cw} - 2x_l - 4x_f - 5.2x_c$$

$$y_{dw} \leq d_{dw}$$

$$y_{tw} \leq d_{tw}$$

$$y_{cw} \leq d_{cw}$$

$$8y_{dw} + 6y_{tw} + y_{cw} - x_l \leq 0$$

$$4y_{dw} + 2y_{tw} + 1.5y_{cw} - x_f \leq 0$$

$$2y_{dw} + 1.5y_{tw} + 0.5y_{cw} - x_c \leq 0$$

Formal concepts

- $\alpha = 0.5 \rightarrow$ we maximize the 50% worst profit

Strategy					F)
Resource quantities	Lumber				600
	Finishing				350
	Carpentry				150
DEMAND		Low	Medium	High	
Production quantities	Desk	50	75	75	
	Tables	20	0	0	
	Chairs	40	0	0	
Profit (\$)		1020	1320	1320	
Expected profit (\$)		1230			

Formal concepts

- $\alpha = 0.5 \rightarrow$ we maximize the 50% worst profit

Strategy	A)	B)	C)	D)	E)	F)
Profit (low demand)	1124	-3618	-11750	-4035	-650	1020
Profit (medium demand)	1124	3982	-1900	3765	2750	1320
Profit (high demand)	1124	3982	7450	4165	2750	1320
Expected Profit (\$)	1124	1702	-2050	1545	1730	1230

Tradeoff between expected profit and risk



Thanks for your attention!